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WAVELET-BASED INDICATORS FOR RESPONSE SURFACE MODELS IN DAMAGE IDENTIFICATION OF STRUCTURES

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Abstract. *In this paper, wavelet energy damage indicator is used in response surface methodology to identify the damage in simulated filler beam railway bridge. The approximate model is addressed to include the operational and surrounding condition in the assessment. The procedure is split into two stages, the training and detecting phase. During training phase, a so-called response surface is built from training data using polynomial regression and radial basis function approximation approaches. The response surface is used to detect the damage in structure during detection phase. The results show that the response surface model is able to detect moderate damage in one of bridge supports while the temperatures and train velocities are varied.*

1 INTRODUCTION

Damage in civil engineering structures can be defined by changes of structural properties that lead to degradation of performance. A robust approach to identify structural damages is by analyzing the vibration responses. The degradation of material or structural properties lead to change in dynamics properties. However, changes in dynamics properties could be the outcomes of environmental condition changes. The structural health monitoring (SHM) systems will not be accepted in practical applications unless robust techniques are developed to explicitly account for environmental and operational conditions, [7].

Response surface methodology (RSM) approach seems able to take into account all damaged and environmental or operational factors that has significant effects on dynamics response of structure. However, the capability of meta models to identify damage is also depend on damage indicator. Low sensitivity damage indicator causes identification procedures more difficult. The difference between damage and non-damage response is too small and can be hidden by noise, model error, or minor parameters that are not included in the models.

Wavelet transform is well known powerful tool in signal analysis. Wavelet reveals hidden information in signals. Instead of wavelet package and wavelet entropy that have been suggested by many researcher e.g. [6, 8, 9], total wavelet energy can be an alternative indicators. The alteration of dynamics response signals due to damage is indicated by the change of energy distribution over frequency sub bands.

In this paper, the response surface methodology was applied to numerical simulated train passage over a filler beam railway bridge. The dynamic response was computed using Newmark method while the damping matrix was constructed using Rayleigh method. The temperature and train speed were chosen as the environmental and operational variables respectively. Wavelet energy was employed as damage indicator while the polynomial regression model and radial basis function approximation were used to construct the surrogate model.

2 WAVELET DAMAGE INDICATOR

2.1 Wavelet Analysis

The history of wavelet transform was started in the 1909 when Alfred Haar introduced rectangular basis function. However the term wavelet was firstly used by Jean Morlet to describe the resulting waveforms of varying window width in short time fourier transform, [1]. The theoretical formulation of wavelet transform was first proposed by Jean Morlet and Alex Grossmann, [15]. The important breakthrough of wavelet analysis emerged in late 1990s. Ingrid Daubechies introduced a so-called Daubechies wavelet bases and Stephane Mallat proposed a general method to construct wavelet bases. The theory of wavelet transform described in many literatures e.g. [10, 12]. A short summary of wavelet analysis is presented here.

The wavelet transform of a function $f(t)$ is written as:

$$W_{\psi}^{f}(a,b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right) \mathrm{d}t \tag{1}$$

The function ψ is often called "mother wavelet" where its value can be real or complex. In this application, ψ is assumed to be real, otherwise the complex conjugate should be introduced to the equation. In the equation (1), it is obvious that the function $f(t)$ is multiplied by a function of two variables (a,b) which is shown in equation (2).

$$\psi_{a,b}(t) = |a|^{-1/2}\,\psi\left(\frac{t-b}{a}\right) \tag{2}$$

The term *wavelets* is used to the function $\psi_{a,b}$, which is actually the dilated (stretched or compressed) and translated versions of *mother wavelet* ψ. a dan b are called scaling parameter and translation parameter respectively. Several types of wavelet families have been known such as Haar, Mexican Hat, Morlet, Meyer, Daubechies etc. Figure (1) shows mother wavelet examples.

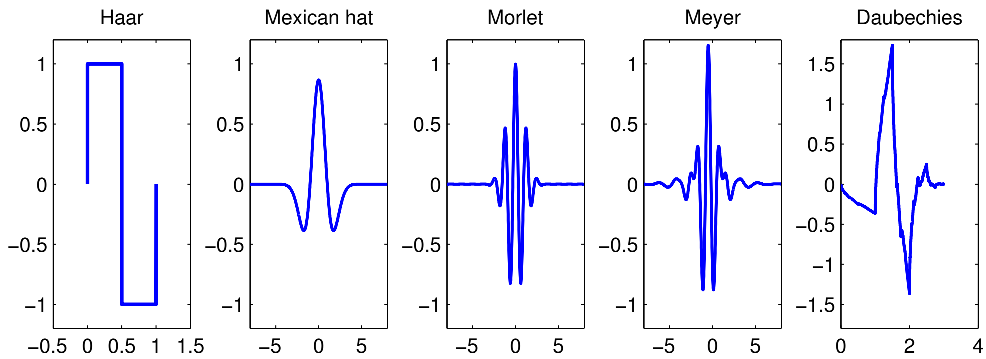

Figure 1: mother wavelet

For discrete wavelet transform, the parameter a and b in equation (2) become discrete and are chosen to be a constant, where $a_m = a_0^m$ and $b_{m,n} = nb_0a_0^m$. $m, n \in \mathbb{Z}$ and $a_0 > 1, b_0 > 0$, [12]. By substituting these constants, the equation (2) becomes:

$$\psi_{m,n} = a_0^{-\frac{m}{2}}\,\psi\left(a_0^{-m}t - nb_0\right) \tag{3}$$

A well-known group of discrete wavelet is given by dyadic wavelet. They are formed by setting $a_0 = 2$ and $b_0 = 1$. By considering these values, the equation (3) can be written as:

$$\psi_{m,n} = 2^{-\frac{m}{2}}\,\psi\left(2^{-m}t - n\right) \tag{4}$$

The numerical implementation of discrete wavelet transform is done by means of the fast wavelet transform (FWT) which is a set of algorithm developed by [14]. The algorithm is based on multiresolution analysis. A short description of wavelet transform is presented here.

A signal f in the subspace V_{-1} in $L^2(\mathbb{R})$ is separated into a high and low frequency part. The low frequency part is the projection $\mathcal{P}_0 f$ onto a lower space V_0. The complement is the projection $\mathcal{Q}_0 f$ into W_0.

$$f = \mathcal{P}_0 f + \mathcal{Q}_0 f, \quad V_{-1} = V_0 \oplus W_0 \tag{5}$$

Therefore a signal f in $L^2(\mathcal{R})$ can be described by the following decomposition:

$$f = \mathcal{P}_M f + \sum_{k=m+1}^{M} \mathcal{Q}_k f, \quad V_m = V_M \oplus \bigoplus_{k=m+1}^{M} W_k \tag{6}$$

which is graphically shown in figure (2).

$$
\begin{tikzcd}
L^2(\mathbb{R}) \cdots \arrow[r] & V_{-1} \arrow[r, "\mathcal{P}_0"] \arrow[dr, "\mathcal{Q}_0"'] & V_0 \arrow[r, "\mathcal{P}_1"] \arrow[dr, "\mathcal{Q}_1"'] & V_1 \cdots \arrow[r] & \{0\} \\
& & \mathcal{W}_0 & \mathcal{W}_1 &
\end{tikzcd}
$$

Figure 2: Scheme of a multi-scale analysis

The multi-scale analysis in the context of orthogonal wavelet transformation assumes the existence of scaling function φ.

$$\varphi_{m,n} = 2^{-\frac{m}{2}} \varphi \left(2^{-m} t - n\right) \tag{7}$$

The scaling function φ satisfies the scaling condition;

$$\varphi(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} a_k \varphi(2t - k), \quad a_k \in \mathbb{R} \tag{8}$$

Based on such a scaling function φ a mother wavelet ψ can be written:

$$\psi(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} b_k \varphi(2t - k), \quad b_k \in \mathbb{R} \tag{9}$$

φ and ψ hold properties,

$$\int_{-\infty}^{\infty} \varphi(t) dt = 1, \quad \int_{-\infty}^{\infty} \psi(t) dt = 0 \tag{10}$$

a_k and b_k follow the conditions,

$$\sum_{k \in \mathbb{Z}} a_k = \sqrt{2}, \quad \sum_{k \in \mathbb{Z}} b_k = 0 \tag{11}$$

By using fast wavelet transform, a signal $f \in V_0 \subset L^2(\mathbb{R})$ defined by equations(5) and (6) can be decomposed as:

$$f(t) = \sum_{k \in \mathbb{Z}} C_{M,k} \varphi_{M,k} + \sum_{m=1}^{M} \sum_{k \in \mathbb{Z}} D_{m,k} \psi_{m,k} \tag{12}$$

Where $C_{m,n}$ and $C_{m,n}$ are approximation coefficients and detail coefficients respectively, which are calculated using the following equations:

$$C_{m,n} = \sum_{k \in \mathbb{Z}} a_{k-2n} C_{m-1,k}, \quad D_{m,n} = \sum_{k \in \mathbb{Z}} b_{k-2n} C_{m-1,k} \tag{13}$$

The algorithm fast wavelet transform based on multi resolution analysis that proposed by [14] has been implemented in SLang software Package, [3] and was used in this study.

2.2 Wavelet Energy Damage Indicator

In signal processing, the energy of a given signal x(t) is defined as :

$$E = \int_t |x(t)|^2 \, \mathrm{d}t \tag{14}$$

Wavelet also has orthonormal basis, therefore the concept of energy in signal processing can also be applied in wavelet. Based on equation (12), the total energy of the decomposition signal up to level M can be calculated from equation (15).

$$\Pi_0 = \sum_k 2^M C_{M,k}^2 + \sum_{m=1}^{M} \sum_k 2^m D_{m,k}^2 \tag{15}$$

C and D indicate the approximation and details of respective wavelet decomposition. The factor 2^m guarantees the energy conservation at each level. Consequently, the absolute wavelet energy of the approximation and detail of a level m is given by equation (16) and (17) respectively.

$$\Pi_{C,m} = 2^m \sum_k C_{m,k}^2 \tag{16}$$

$$\Pi_{D,m} = 2^m \sum_k D_{m,k}^2 \tag{17}$$

3 RESPONSE SURFACE METHODOLOGY

3.1 Overview

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes, [4]. The RSM emerged in 1951 when G. E. P. Box and K. B. Wilson proposed empirical model to study the relationship between some variables in chemical experimental study, [16]. Three decades later, this approach was also applied in numerical or computer experiments. RSM has been applied to optimize the high-speed mass transport, [11]. In [5] and [13], the RSM approach was used for reliability analysis. This paper presents the application of RSM in structural health monitoring (SHM) systems.

3.2 Damage Detection Response Surface

In damage identification, the RSM is used to take into account all variables that contribute the variation of structural response including the operational and environmental conditions. The procedure is split into two stages, the *Learning Phase* and the *Detection Phase* which are elaborated in the following subsections.

3.2.1 Learning Phase

The learning phase is commenced by a basic assumption that no damage is present in the structure. The vibration response of structure is measured including its corresponding operational and surrounding condition. The measured or simulated response signal is further processed to obtain the damage indicator values. The indicator values will be used to train the approximation model. Therefore, this phase is also known as *Training Phase*. The model is called *Reference Surface*, the benchmark of healthy structure responses in various environmental and operational conditions. Figure (3) illustrates the approximation models and its observation points.

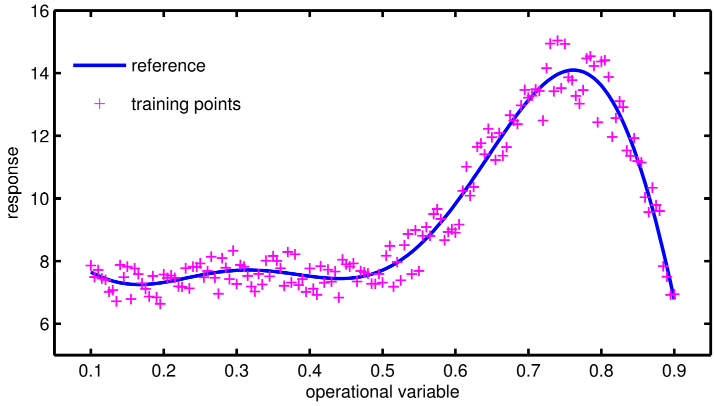

Figure 3: Training phase: the reference surface is built from observation points using scattered data approximation

In numerical simulation, the training points can be set by using design of experiments (DoE) such as full factorial design, fractional factorial design or latin hypercube sampling. Both stationer and non-stationer data can be generated. However, in real application the stationer space filling is generally not possible. The data spread irregularly because most of the variation of environmental or operational variables are uncontrollable. Furthermore, it also possible to have support points that are concentrated in certain regions. In this case, the effect of influence radius in approximation with weighting function should be carefully observed. Poor approximation can be achieved in the neighboring area of dense support points.

For model selection and validation, the observation data is split into two parts, the training and testing data sets. The first is used to fit the model while the second is used to compute the model error. It is importance to assess the model quality in non-sampled region. The simple way to assess the quality of approximation model is by using coefficient determination (R^2). Better model has higher R^2. Another approach that is also very popular is called cross validation. Furthermore, the sensitivity analysis is employed to improve the surrogate model. Some variables possibly do not have significant impact on model response. Therefore, omitting these variables is useful to reduce model complexity and improve model quality.

Several methods are available to construct reference surface from scattered points. Polynomial regression is very famous because of its simplicity. However, high order polynomials are required to approximate a surface with many peaks and troughs. Combination of many variables and higher order increase the complexity in model selection. Furthermore, high order polynomial order can lead to over fitting in noise data. More local representative is obtained by weighting method such as radial basis function or moving least square. Artificial neural networks can also be an option in model approximation. Polynomial and radial basis function is applied in this study. More detail elaboration about these method is described in section (3.3).

3.2.2 Detection Phase

The detection phase is started after the reference surface is built. The damage identification is done by comparing damage indicator value from actual measurement to the reference surface. Significant deviation from reference indicates the presence of damage. Figure (4) illustrates the responses of damage structures and it respective benchmark.

The capability of the reference surface to detect damages is depend on the sensitivity of damage indicator. Less sensitivity indicator are not able to support the response surface model to identify the damage. The reason is because the distinction between damaged and non-damaged response is too narrow. In real application this small discrepancy is potentially obscured by noise of measurement or approximation error. Another requirement for damage indicator is stability with respect to damage severity. It means no sudden change of gradient sign when the damage becomes more severe.

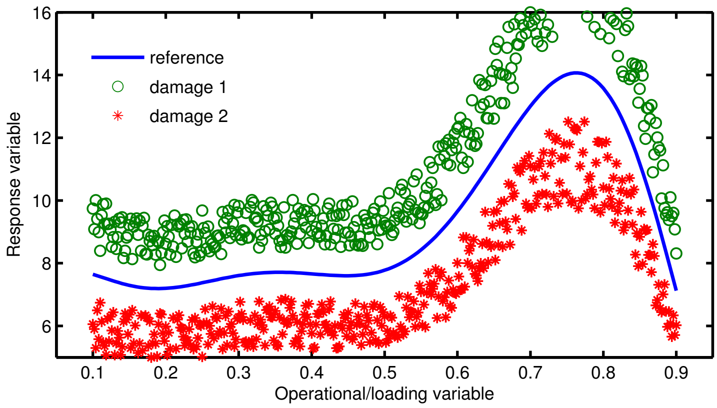

Figure 4: Detection phase: the actual response is compared to reference surface

Basically, RSM approximation approach is very suitable for damage detection of structures after extreme loading event. The structural condition can immediately assess by using ambient excitation from vehicles, winds, or pedestrian steps. However, application for long term monitoring also gives a great advantage. The RSM damage identification can be used to monitor the degradation of structural performance or damage growth. Therefore this phase also known as *monitoring phase*.

3.3 Scattered Data Approximation

As mentioned earlier, several methods are available to construct the reference surface from scattered data. Polynomial and radial basis function are used in this study. Short review about these approaches is described in the following sub subsection.

3.3.1 Polynomial Model Regression

Basically, the polynomial approximation of a function is similar to a Taylor series expansion of function f after m+1 terms, Box:1987. The flexibility of the estimation function increases as a new higher polynomial order is included in the model. A polynomial approximation of a function $y = f(x)$ of order m is written in equation (18).

$$y = \beta_0 + \beta_1 + \beta_2 x^2 + \cdots + \beta_m x^m + \varepsilon \tag{18}$$

In a matrix form the equation (18) can be written as (19), where X is known as *Vandermonde matrix*.

$$y = X\beta + \varepsilon \tag{19}$$

$$\begin{Bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^m \\ 1 & x_2 & x_2^2 & \dots & x_2^m \\ 1 & x_3 & x_3^2 & \dots & x_3^m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^m \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{Bmatrix} + \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \tag{20}$$

The estimation variables β are solved through least square solution of equation (20), which give $\beta = X^{+}y$, where $X^{+} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$ is known as *Moore-Penrose pseudo-inverse*.

The polynomial model approximation can also apply to multivariable systems by expanding the equation (18). For second order, multivariable polynomial model with interaction terms has general form:

$$\begin{aligned} y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \beta_{11} x_1^2 + \cdots + \beta_{kk} x_k^2 \\ + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \cdots + \beta_{k-1,k} x_{k-1} x_k + \varepsilon \end{aligned} \tag{21}$$

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum \sum_{i<j=2}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{22}$$

3.3.2 Radial Basis Function Approximation

The general equation of RBF interpolation in space dimension *s* can be written as:

$$F(x) = \sum_{k=1}^{N} c_k \varphi(\| x - x_k \|), \quad x \in \mathbb{R}^s \tag{23}$$

Where, φ is basis function. $(\| x - x_k \|)$ is a matrix of Euclidean distance, for n points this matrix becomes:

$$\begin{Bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{Bmatrix} = \begin{Bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{Bmatrix} \begin{bmatrix} \varphi(\| x_1 - x_1 \|) & \varphi(\| x_1 - x_2 \|) & \dots & \varphi(\| x_1 - x_n \|) \\ \varphi(\| x_2 - x_1 \|) & \varphi(\| x_2 - x_2 \|) & \dots & \varphi(\| x_2 - x_n \|) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi(\| x_n - x_1 \|) & \varphi(\| x_n - x_2 \|) & \dots & \varphi(\| x_n - x_n \|) \end{bmatrix} \tag{24}$$

Coefficient c_k are obtained by solving the linear equation system in (23). The solution of this equation is unique if the Euclidean distance matrix is non-singular. Certain type of basis function can be used to develop a positive definite weighting matrix. Several basis function can be used in RBF e.g. the truncated power function, multiquadratic or inverse multiquadratic. A widely used basis function is *Gaussian fuction* as described in equation (25). This type of basis function also used in this study.

$$\varphi(r) = e^{-(\epsilon r)^2} \tag{25}$$

The shape parameter ϵ in equation (25) is used to adjust the local influence of support points. Figure (5) illustrates how a shape parameter has a profound influence on how flat or peak (localize) is the basis function. Smaller value of ϵ that also means larger variance results a flatter surface while larger value of ϵ tend to have more peaked plot. Furthermore, it affects the accuracy and numerical stability of the approximation.

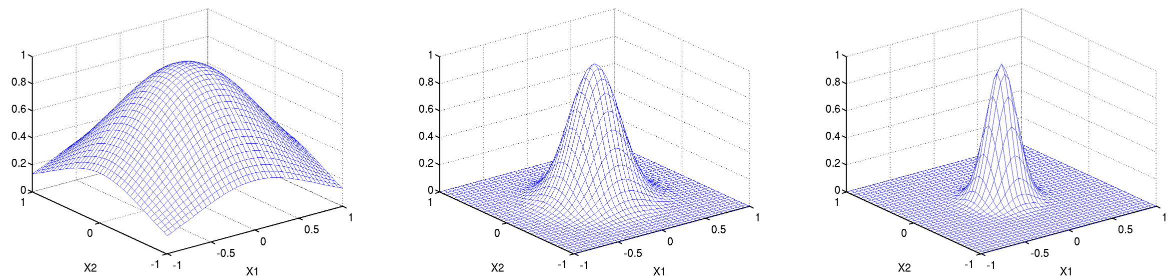

Figure 5: Shape parameter ϵ is used to adjust the local influence. Shape parameter in gaussian basis function, ϵ = 1 (left), ϵ = 3 (middle) and ϵ = 5 (right).

4 CASE STUDY AND RESULTS

4.1 Simple Supported Steel Beam

Simple supported steel beam is modeled using finite element package to observe the sensitivity of wavelet energy damage indicator. The beam is a modification of standard IPE 80 section. The additional 40 mm thickness plate is attached to its bottom flanges. The section with extra plate is considered as non-damaged structure. The damage scenario is carried on by turning back the section to its original section. The beam is illustrates in figure (6). The natural frequencies and their corresponding shape of vertical bending mode are shown in figure (7).

The beam was excited by single impulse in vertical direction. The responses signals were monitored in 5 locations as indicates in figure (7). The extracted vertical acceleration signals were decomposed using wavelet. The energies content in the decomposed signals were calculated using equations (15, 16, and 17).

The performance of eigen frequencies and modal displacement to identify the damage is summarized in figure (8). There are no significant differences in frequencies from damaged and non-damaged scenarios. Only 0.3 % approximately of frequency changes is obtained from these two scenarios. More clear differences are shows by modal displacement amplitude. However

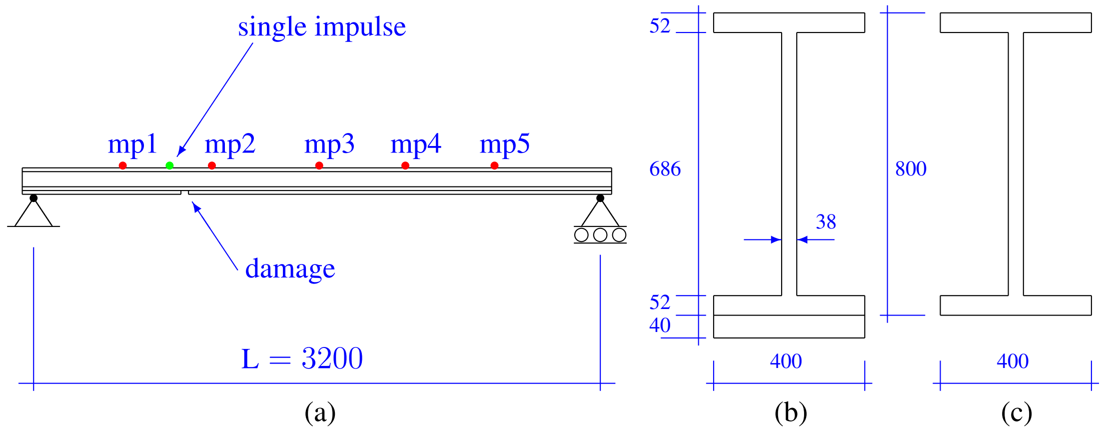

Figure 6: (a) Simple supported steel beam model, (b) Non-damaged section, (c) Damaged section.

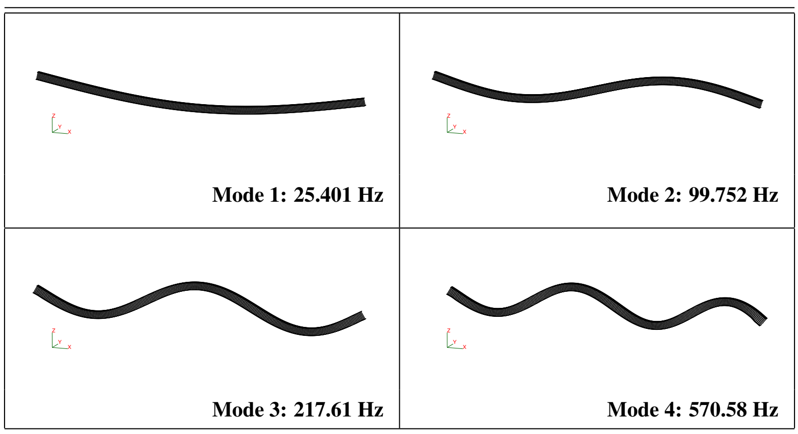

Figure 7: Vertical bending mode shapes of simulated simple supported beam

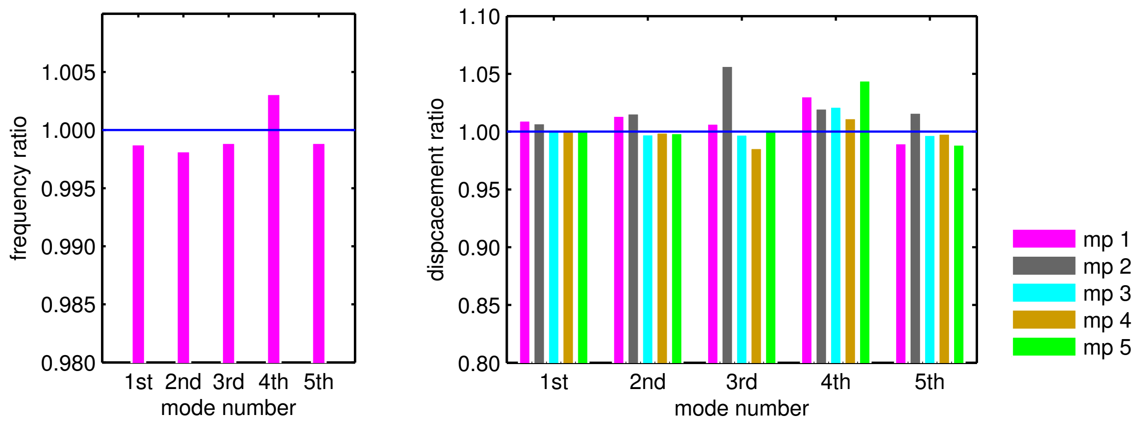

Figure 8: frequencies and modal displacement comparison

this trend appears in higher frequencies mode shapes. In practice, high frequency mode shape is difficult to be extracted.

In general, wavelet energy show much better result compared to frequencies or modal displacement amplitude. The difference between wavelet energy in damaged and non-damage scenario exceed 3 % in all extracted signals. Therefore this indicator is more suitable to be applied in RSM damage identification. The energy ratio between damage and non-damage structure is shown in figure(9).

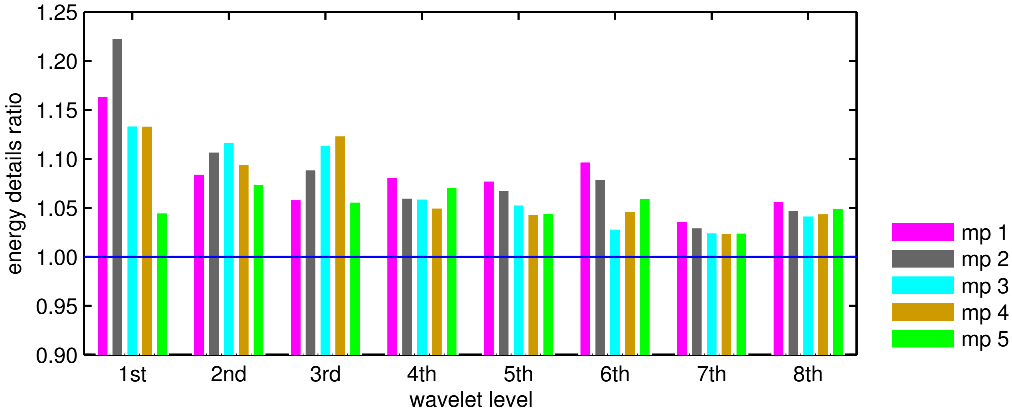

Figure 9: comparison of wavelet energy details in damaged and nondamaged structure

4.2 Railway Bridge

The response surface model for damage detection was applied to filler beam bridge through numerical simulation. The bridge has 24.60 m span and 5.39 m width. The main structure consists of girder and deck slab which are modeled by beam and shell elements respectively. The 3D spring elements are used to model the elastomeric bearings which are directly support the steel I-shape main girders. The similar elements also are used to model the ballast, where five spring elements are allocated to support each sleeper. Figure (10) shows the finite element model of the bridge. Three of the lowest eigen frequencies of the bridge model are 3.63 Hz, 5.53 Hz, and 9.18 Hz.

A collective of moving loads was built and shifted for each time step to develop the dynamic excitation. This collective load represents a series of ICE3 train, one load for each wheel. It means 64 vertical point loads were applied to the structure with sampling rate 500 Hz. Possible effects due to rail-roughness, train-bridge-interaction or wheel irregularities were neglected for simplification.

The speed of the loading train and temperature were chosen as operational and environmental condition respectively. The train speed varies from 200 to 300 km/h. The temperature is assumed to affect the elastomeric material only. The temperature effects model is adopted from [2]. The relation between temperature and shear modulus is described as $G = 4.06574 - 0.0271153T - 0.005503455T^2, \quad -5°C \leq T \leq 21°C$, where G and T are shear modulus and temperature respectively. The temperature-shear modulus model was obtained from long term dynamics experimental test of the similar bridge.

Full factorial design was adopted for design of experiments to generate 42 regular training points. The same approach was also applied to obtain 30 testing points for model selection and

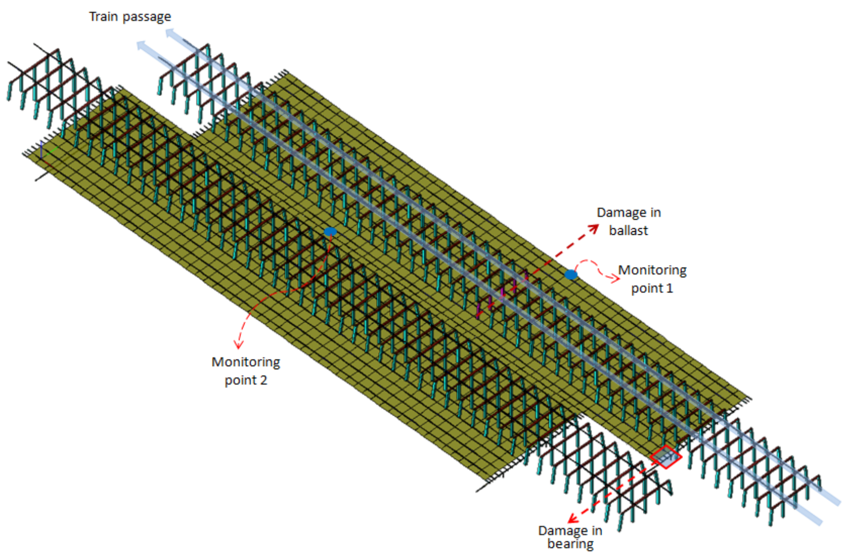

Figure 10: Finite element model of the bridge.

validation. The support points and its correspond response surface that was generated using *polynomial model* is shown in figure (11a). It shows that the 1st level wavelet energy fitted well by polynomial model. The response surface generated by RBF is show in figure (11b).

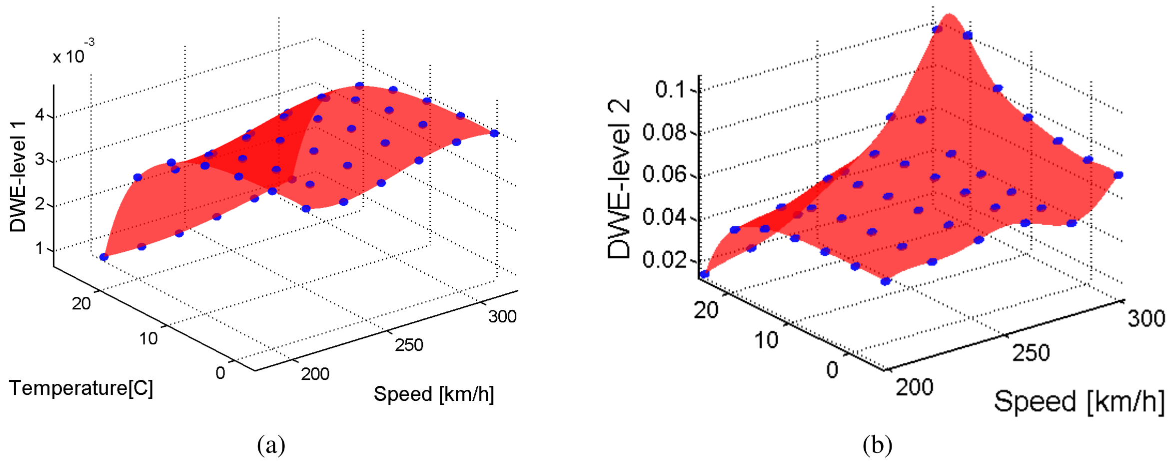

Figure 11: (a) Reference surface of wavelet energy level 1 generated using polynomial regression. (b) Reference surface of wavelet energy level 2 generated using radial basis fuction approximation.

The damage is prescribed by reducing 50 % stiffness of one elastomeric support. 5 variation of train speed and surrounding temperature are selected using *Latin Hypercube Sampling* method. The level 2 wavelet energy of the testing points are plot together with its reference in figure(12a). A clear distinction of damage and non-damaged wavelet energy is obtained in 3 points. The energy ratio of these 3 points changed up to 25 % while only 10 % changes is monitored from the rest 2 points. However, these 2 points indicates much better differentiation in the next wavelet energy level. Therefore, the change of wavelet energy should be observed in all level in order to have comprehensive assessment of structural condition. The comparison of wavelet energy in the damage and healthy condition in all level of wavelet is presented in figure (12b).

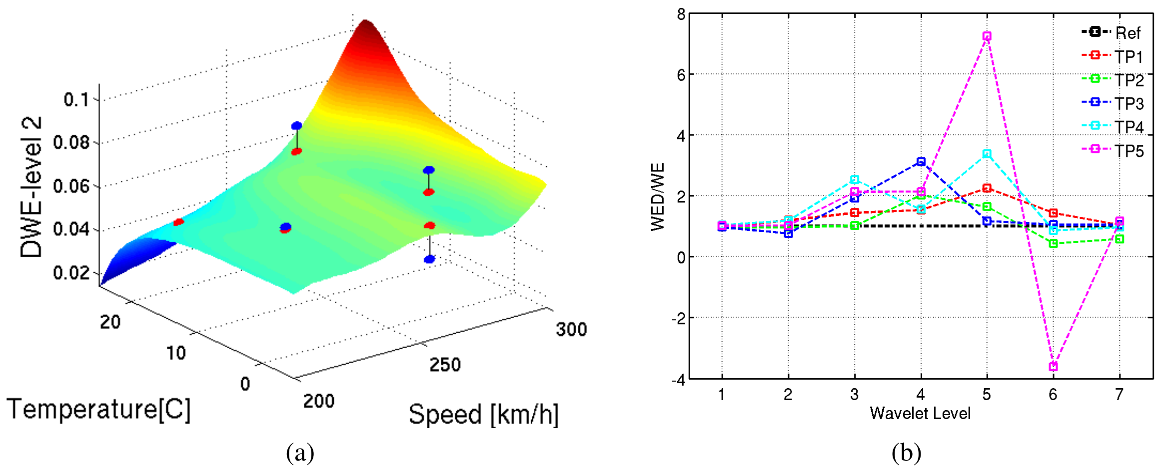

Figure 12: Response surface method for damage identification. (a) Response of damage structure and its reference at level 2. (b) Comparison wavelet energy in all level

5 CONCLUSIONS

The paper describes an approximation procedure to assess the structural health in assorted operational and environmental condition by using wavelet energy as response indicator. The response surface methodology is a potential approach to deal with damage detection problem in a situation where the non-damaged variables also affect the data response. A quite sensitive damage indicator is also very substantial to increase the capability of the method.

From the simulation results, wavelet energy is a good candidate for an indicator in combination with response surface damage identification. Its sensitivity allows to distinguish the alteration between two scenarios of numerical simulation. However, more effort is needed to be spent in observation of each wavelet level because the sensitivity is different from one to other level. As the signal becomes longer, more level will be acquired from the decomposition.

Both polynomial and RBF model show a good performance for model approximation in this study. However, the polynomial models are not able generate a good approximation in higher level of wavelet energy. The complexity of searching the best polynomial model increase significantly as new variables are introduced to the surrogate model while in RBF new variables only add new term in calculation of Euclidean distances. Therefore, the RBF model is more suitable as the number of variable increase.

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